

Superbrane Actions and Geometrical Approach *

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Abstract

We review a generic structure of conventional (Nambu–Goto and Dirac–Born–Infeld–like) world-volume actions for the superbranes and show how it is connected through a generalized action construction with a doubly supersymmetric geometrical approach to the description of super-p-brane dynamics as embedding world supersurfaces into target superspaces.

During last years Dmitriy Vasilievich Volkov actively studied geometrical and symmetry grounds underlying the theory of supersymmetric extended objects and we are happy to have been his collaborators in this work. One of the incentives for this research was to understand the nature of an important fermionic κ -symmetry of the target-superspace (or Green–Schwarz) formulation of the superparticles and superstrings with the aim to resolve the problem of its infinite reducibility, to relate the Green–Schwarz and Ramond–Neveu–Schwarz formulation of superstrings already at the classical level and to attack the problem of covariant quantization of superstrings. The κ -symmetry was conjectured to be a manifestation of local extended supersymmetry (irreducible by definition) on the world supersurface swept by a super-p-brane in a target superspace. This was firstly proved for $N = 1$ superparticles in three and four dimensions [1] and then for $N = 1$, $D = 6, 10$ superparticles [2], $N = 1$ [3], $N = 2$ [4] superstrings, $N = 1$ supermembranes [5] and finally for all presently known super-p-branes

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[6, 7, 8] in all space–time dimensions where they exist. In [9] a twistor transform was applied to relate the Green–Schwarz and the Ramond–Neveu–Schwarz formulation.

The approach to describing the super–p–branes in this way is called the doubly supersymmetric geometrical approach, since it essentially exploits the theory of embedding world *supersurfaces* into target *superspaces*. Apart from having clarified the geometrical nature of κ –symmetry and having made a substantial impact on the development of new methods of superstring covariant quantization (see [10, 11] and references therein), the doubly supersymmetric approach has proved its power in studying new important class of super–p–branes (such as Dirichlet branes [12] and the M–theory five–brane [13]) for which supersymmetric equations of motion were obtained in the geometrical approach [8] earlier than complete supersymmetric actions for them were constructed by standard methods [14, 15]. Thus, a problem arises to relate the super–p–brane equations obtained from the action with the field equations of the doubly supersymmetric geometrical approach, and to convince oneself that they really describe one and the same object. To accomplish this goal one should reformulate the action principle for the super–p–branes such that it would yield the embedding conditions of the geometrical approach in the most direct way. For ordinary super–p–branes such an action has been proposed in [7]. The construction is based on generalized action principle of the group–manifold (or rheonomic) approach to superfield theories [16]. D. V. Volkov considered this approach as the most appropriate for implementing geometry of the supersymmetric extended objects into the description of their dynamics.

In this contribution we would like to review basic elements of the generalized action construction and to show that it is also applicable to the Dirichlet branes [17] and, at least partially, to the M–theory five–brane (M–5–brane), thus allowing one to establish the relation between the formulations of [14, 15] and [8, 18].

On the way of reconstructing the super–p–brane actions we shall answer another question connected with their κ –symmetry transformations, namely, a puzzling fact that the κ –transformation of a “kinetic” part of the conventional super–p–brane actions is the integral of a $(p + 1)$ –form which compensates the κ –variations of a Wess–Zumino term of the actions. This puzzle is resolved in a formulation where the entire action of a super–p–brane is the integral of a differential $(p + 1)$ –form in the worldvolume of the brane [6, 7]. To construct such an action one uses auxiliary harmonic [19] or twistor–like variables which enable to get an irreducible realization of the κ –transformations (see [1, 10, 20] and references therein for superparticles, superstrings and type *I* super–p–branes). We shall also see that in the case of the D–branes and the M–5–brane this version of the action serves as a basis for the transition to a dual description of these objects.

Consider the general structure of the action for a super–p–brane propagating in a supergravity background of an appropriate space–time dimension (which is specified by a brane scan [21]). We work with actions of a Nambu–Goto (or Dirac–Born–Infeld) type that do not involve auxiliary fields of intrinsic worldvolume geometry as in the Brink–Di Vecchia–Howe–Tucker form [22] of brane actions (see [23, 24] for the BDHT approach to D–branes).

All known super–p–brane actions, except that of the M–5–brane which contains a third term (see below), generically consist of two terms:

$$S = I_1 + I_{WZ} = \int_{M_{p+1}} d^{p+1}x e^{-\frac{p-3}{4}\phi} \sqrt{-\det G_{mn}} + \int_{M_{p+2}} W_{p+2}. \quad (1)$$

The symmetric part g_{mn} of the matrix $G_{mn} \equiv g_{mn} + \mathcal{F}_{mn}$ in the first term of (1) describes a super–p–brane worldvolume metric induced by embedding into a target superspace which is

parametrized by bosonic coordinates $X^{\underline{m}}(x)$ ($\underline{m} = 0, 1, \dots, D-1$) and fermionic coordinates $\Theta^{\underline{\mu}}(x)$ ($\underline{\mu} = 1, \dots, 2^{\lfloor \frac{D}{2} \rfloor}$) collectively defined as $Z^{\underline{M}} = (X^{\underline{m}}, \Theta^{\underline{\mu}})$. The worldvolume itself is parametrized by small x^m ($m = 0, \dots, p$) with not underlined indices. $\phi(Z)$ is a background dilaton field. Note that there is no such a field in $D = 11$ supergravity.

The antisymmetric part \mathcal{F}_{mn} of G_{mn} , which is absent from ordinary superbranes and nonzero for the D-branes and the M-5-brane, contains the field strength of a gauge field propagating in the brane worldvolume plus the worldvolume pullback of a Grassmann-antisymmetric field of target-space supergravity.

In the case of the D-branes in $D = 10$ the worldvolume field is a vector field $A_m(x)$ [12, 14], the background field is a two-rank superfield $B_{\underline{MN}}(X, \Theta)$, and \mathcal{F}_{mn} has the form

$$\mathcal{F}_{mn}^{(D)} = e^{-\frac{\phi(Z)}{2}} (\partial_m A_n - \partial_n A_m + \partial_m Z^{\underline{N}} \partial_n Z^{\underline{M}} B_{\underline{MN}}). \quad (2)$$

In the case of the M-5-brane the worldvolume gauge field is a self-dual (or chiral) tensor field $A_{mn}(x)$, and the background field is a three-rank superfield $C_{\underline{LMN}}(X, \Theta)$ of $D = 11$ supergravity [13, 15]. The M-5-brane action also contains an auxiliary worldvolume scalar field $a(x)$ [25] whose presence ensures manifest $d = 6$ worldvolume covariance of the model [26, 15]. In this case the antisymmetric matrix takes the form

$$\mathcal{F}_{mn}^{(M)} = \frac{i}{\sqrt{\partial_p a \partial^p a}} H_{mnl}^* \partial^l a(x), \quad H_{mnl} = 6 \partial_{[l} A_{mn]} + \partial_l Z^{\underline{L}} \partial_m Z^{\underline{N}} \partial_n Z^{\underline{M}} C_{\underline{MNL}}, \quad (3)$$

where $*$ denotes Hodge operation, e.g. $H_{mnl}^* = \frac{\sqrt{-g}}{3!} \varepsilon_{mnlpqr} H^{pqr}$.

The second term in (1) is a Wess-Zumino (WZ) term. Generically it is more natural to define it as an integral of a closed differential $(p+2)$ -form over a $(p+2)$ -dimensional manifold whose boundary is the super-p-brane worldvolume. The structure of the WZ term depends on the p-brane considered and (in general) includes worldvolume pullbacks of antisymmetric gauge fields of target-space supergravity and their duals (see [14, 15] for details).

The third term which one must add to the action (1) to describe the M-5-brane dynamics is quadratic in H_{mnl} [15]:

$$I_3 = \int d^6 x \frac{i}{\sqrt{-\partial_p a \partial^p a}} \mathcal{F}_{mn}^{(M)} H^{mnl} \partial_l a(x). \quad (4)$$

In this case the action (1) plus (4) is invariant under the local symmetries [25, 26]

$$\delta A_{mn} = \frac{\varphi(x)}{2(\partial a)^2} (H_{mnp} \partial^p a - \mathcal{V}_{mn}), \quad \delta a(x) = \varphi(x) \quad (5)$$

and

$$\delta A_{mn} = \partial_{[m} a(x) \varphi_{n]}(x), \quad \delta a(x) = 0 \quad (6)$$

where

$$\mathcal{V}^{mn} \equiv -2 \sqrt{\frac{(\partial a)^2}{g}} \frac{\delta \sqrt{-\det(G_{pq})}}{\delta \mathcal{F}_{mn}},$$

and φ and φ_m are local gauge parameters. The local symmetry (5) allows one to gauge the field $a(x)$ away at the expense of manifest Lorentz invariance of the M-5-brane action and the local symmetry (6) is needed to ensure the self-duality condition for A_{mn} . These local symmetries

are, in some sense, a bosonic analog of the fermionic κ -symmetry (see below) whose gauge fixing also results in the loss of Lorentz covariance.

The action (1) (plus (4) in the case of the M-5-brane) is invariant under the following κ -transformations of the worldvolume fields

$$\begin{aligned} i_\kappa E^\alpha &\equiv \delta_\kappa Z^{\underline{M}} E_{\underline{M}}^\alpha = \kappa^\alpha, & i_\kappa E^a &= 0, \\ \delta_\kappa g_{mn} &= -4iE_{\{m}\Gamma_{n\}} i_\kappa E, & (7) \\ \delta_\kappa \mathcal{F}^{(D)} &= i_\kappa dB_{(2)}, & \delta_\kappa H &= i_\kappa dC_{(3)}, & \delta_\kappa a(x) &= 0, \end{aligned}$$

where

$$E^{\underline{A}} = (dZ^{\underline{M}} E_{\underline{M}}^{\underline{a}}(X, \Theta), dZ^{\underline{M}} E_{\underline{M}}^{\underline{a}}(X, \Theta)) \quad (8)$$

are target-space supervielbeins pulled back into the worldvolume. They define the induced metric

$$g_{mn} = \partial_m Z^{\underline{M}} \partial_n Z^{\underline{N}} E_{\underline{N}}^{\underline{a}} \eta_{\underline{a}\underline{b}} E_{\underline{M}}^{\underline{b}}, \quad (9)$$

and i_κ denotes the contraction of the forms with the κ -variation of $Z^{\underline{M}}$ as written above. The Grassmann parameter $\kappa^\alpha(x)$ of the κ -transformations satisfies the condition

$$\kappa^\alpha = \kappa^\beta \bar{\Gamma}_{\underline{\beta}}^{\underline{\alpha}}, \quad (10)$$

where $\bar{\Gamma}$ is a traceless matrix composed of the worldvolume pullbacks of target-space Dirac matrices and the tensor \mathcal{F}_{mn} such that $\bar{\Gamma}^2 = 1$. The form of $\bar{\Gamma}$ is specific for a p -brane considered and reflects the structure of the WZ term [14, 15].

Eq. (10) reads that not all components (in fact only half) of κ^α are independent, which causes the (infinite) reducibility of the κ -transformations. If one tries to get an irreducible set of κ -parameters in this standard formulation, one should break manifest Lorentz invariance of the models. The geometrical approach considered below provides us with a covariant way of describing independent κ -transformations.

For all super- p -branes the κ -variation (7) of the Wess-Zumino term is (up to a total derivative) the integral of a $(p+1)$ -form

$$\delta_\kappa I_{WZ} = \int_{M_{(p+1)}} i_\kappa W_{(p+2)}. \quad (11)$$

For the complete action to be κ -invariant the WZ variation must be compensated by the variation of the NG or DBI-like term (and the term (4)). Thus, though these parts of the action are not the integrals of differential forms, their κ -variations are. To explain this puzzling fact it is natural to look for a formulation where the entire action is the integral of a $(p+1)$ -form. When we deal with ordinary super- p -branes, for which $\mathcal{F}_{mn} = 0$, this can be easily done, since (apart from the presence of the dilaton field) the NG term in (1) is the integral volume of the world surface and can be written as the worldvolume differential form integral

$$I_1 = \int_{M_{(p+1)}} \frac{1}{(p+1)!} E^{a_0} \wedge E^{a_1} \wedge \dots \wedge E^{a_p} \epsilon_{a_0 a_1 \dots a_p}, \quad (12)$$

where $E^a = dx^m E_m^a(x)$ is a worldvolume vielbein form. Since we consider induced geometry of the worldvolume, E^a is constructed as a linear combination of the target-space supervielbein vector components (8)

$$E^a = dx^m \partial_m Z^{\underline{M}} E_{\underline{M}}^{\underline{b}} u_{\underline{b}}^a(x). \quad (13)$$

$u_b^a(x)$ are components (vector Lorentz harmonics along the worldvolume) of an $SO(1, D-1)$ -valued matrix

$$u_{\underline{b}}^{\underline{\tilde{a}}} = (u_{\underline{b}}^a, u_{\underline{b}}^i) \quad a = 0, \dots, p \quad i = p+1, \dots, D-1, \quad (14)$$

$$u_{\underline{a}}^{\underline{\tilde{c}}} u_{\underline{\tilde{c}}}^{\underline{b}} = \delta_{\underline{a}}^{\underline{b}}, \quad u_{\underline{\tilde{a}}}^{\underline{c}} u_{\underline{c}}^{\underline{\tilde{b}}} = \delta_{\underline{\tilde{a}}}^{\underline{\tilde{b}}} = \text{diag}(\delta_a^b, \delta^{ij}), \quad (15)$$

The orthogonality conditions (15) are invariant under the direct product of target-space local Lorentz rotations $SO(1, D-1) \times SO(1, D-1)$ acting on u from the left and right, while the splitting (14) breaks one $SO(1, D-1)$ (tilded indices) down to its $SO(1, p) \times SO(D-p-1)$ subgroup, which form a natural gauge symmetry of the p -brane embedded into D -dimensional space-time.

Surface theory tells us that (14) can always be chosen such that on the world surface

$$E^i = dZ^M E_{\underline{M}}^b u_b^i(x)|_{M_{p+1}} = 0, \quad (16)$$

i.e. orthogonal to the surface.

Dynamically one derives Eq. (16) from the action (12) by varying it with respect to the auxiliary variables u_b^a and taking into account the orthogonality condition (15).

In view of (13), (16) and (15) we see that the expression (9) for the induced metric reduces to $g_{mn} = E_m^a E_{an}$. Hence, we can replace the determinant of E_m^a written in (12) with $\sqrt{-\det g_{mn}}$ and return back to the NG form of the super- p -brane action. This demonstrates the equivalence of the two formulations.

Note that only vector components $E^{\underline{a}}$ of the target-space supervielbein (8) enter the action (12) through Eqs. (13). But in target superspace a supervielbein also has components along spinor directions (8) (i.e. $E^{\underline{\alpha}}$). When the supervielbein vector components undergo a local $SO(1, D-1)$ transformation with the matrix (14), the supervielbein spinor components are rotated by a corresponding matrix $v_{\underline{\beta}}^{\underline{\tilde{\alpha}}}(x)$ of a spinor representation of the group $SO(1, D-1)$, the matrices $u_{\underline{b}}^{\underline{\tilde{a}}}$ and $v_{\underline{\beta}}^{\underline{\tilde{\alpha}}}$ being related to each other through the well-known formula (see for instance [10, 20])

$$u_{\underline{b}}^{\underline{\tilde{a}}} \Gamma_{\underline{\alpha}\underline{\beta}}^{\underline{b}} = v_{\underline{\alpha}}^{\underline{\tilde{\delta}}} \Gamma_{\underline{\tilde{\delta}}\underline{\gamma}}^{\underline{\tilde{a}}} v_{\underline{\beta}}^{\underline{\tilde{\gamma}}}, \quad u_{\underline{b}}^{\underline{a}} \Gamma_{\underline{\alpha}\underline{\beta}}^{\underline{b}} = u_b^a \Gamma_{\underline{\alpha}\underline{\beta}}^b - u^{ia} \Gamma_{\underline{\alpha}\underline{\beta}}^i = v_{\underline{\alpha}}^{\underline{\tilde{\delta}}} \Gamma_{\underline{\tilde{\delta}}\underline{\gamma}}^a v_{\underline{\beta}}^{\underline{\tilde{\gamma}}}. \quad (17)$$

The matrix $v_{\underline{\beta}}^{\underline{\tilde{\alpha}}}$ satisfies an orthogonality condition analogous to (15). Thus, it is natural to consider the spinor harmonic variables $v_{\underline{\beta}}^{\underline{\tilde{\alpha}}}$ as independent and $u_{\underline{b}}^{\underline{\tilde{a}}}$ composed of the former. The $SO(1, p) \times SO(D-p-1)$ invariant splitting of v (analogous to (14)) is

$$v_{\underline{\beta}}^{\underline{\tilde{\alpha}}} = (v_{\underline{\beta}}^{\alpha q}, v_{\underline{\beta}}^{\dot{\alpha} \dot{q}}), \quad (18)$$

where $\alpha, \dot{\alpha}$ are the indices of (the same or different) spinor representations of $SO(1, p)$ and q, \dot{q} correspond to representations of $SO(D-p-1)$. The choice of these representations depends on the dimension of the super- p -brane and the target superspace considered and is such that the dimension of the $SO(1, p)$ representations times the dimension of the $SO(D-p-1)$ representations is equal to the spinor representation of $SO(1, D-1)$.

To generalize Eq. (12) to the case of the D-branes one should take into account the presence of the antisymmetric tensor \mathcal{F}_{mn} in (1) as follows:

$$I_1 = \int_{M_{(p+1)}} \left(\frac{1}{(p+1)!} E^{a_0} \wedge E^{a_1} \wedge \dots \wedge E^{a_p} \epsilon_{a_0 a_1 \dots a_p} e^{-\frac{p-3}{2}\phi} \sqrt{-\det(\eta_{ab} + \mathcal{F}_{ab})} \right) \quad (19)$$

$$+Q_{p-1} \wedge [e^{-\frac{1}{2}\phi}(dA - B_{(2)}) - \frac{1}{2}E^b \wedge E^a \mathcal{F}_{ab}] + \int_{M_{p+2}} W_{p+2}$$

where we also included the WZ term, and the worldvolume scalar \mathcal{F}_{ab} is an auxiliary antisymmetric tensor field with tangent space (Lorentz group) indices and Q_{p-1} is a Lagrange multiplier differential form which produces the algebraic equation

$$\mathcal{F}_2^{(D)} \equiv 1/2 E^b \wedge E^a F_{ab} = e^{-\frac{\phi}{2}}(dA - B_2). \quad (20)$$

Eq. (20) relates \mathcal{F}_{ab} to the 2-form $\mathcal{F}_2^{(D)}$ of the original action (1).

In the case of the M-5-brane its action (1) plus (4) is written as an integral of differential forms as follows

$$\begin{aligned} I_1 + I_3 = \int_{M_6} & \left(\frac{1}{6!} E^{a_0} \wedge E^{a_1} \wedge \dots \wedge E^{a_p} \epsilon_{a_0 a_1 \dots a_p} (\sqrt{-\det(\eta_{ab} + \mathcal{F}_{ab})} - \frac{i}{4\sqrt{-v_a v^a}} \mathcal{F}^{ab} H_{abc} v^c) \right. \\ & + Q_3 \wedge [dA_2 - C_3 - \frac{1}{3} E^c \wedge E^b \wedge E^a H_{abc}] \\ & \left. + Q_5 \wedge (da(x) - E^a v_a) + \int_{M_7} W_7. \right. \end{aligned} \quad (21)$$

In (21) $\mathcal{F}^{ab} = \frac{i}{\sqrt{-v_a v^a}} H^{*abc} v_c$, and $H_{abc}(x)$ and $v_a(x)$ are auxiliary worldvolume scalar fields which are expressed in terms of original fields A_{mn} and $a(x)$ (3) upon solving the equations of motion for the Lagrange multiplier forms Q_3 and Q_5 .

The variation of the actions (19) and (21) with respect to the auxiliary fields \mathcal{F}_{ab} , $H_{abc}(x)$ and $v_a(x)$ produces algebraic expressions for the Lagrange multipliers $Q_{(n)}$, which thus do not describe independent degrees of freedom of the models. Note also that, at least for the Dirichlet branes with $p \leq 4$, one can invert the equations for Q_{p-1} in terms of \mathcal{F}_{ab} , express the latter in terms of the former and substitute them in the action. This gives a dual worldvolume description of the D-branes [28, 26, 24]. Thus the actions (19) and (21) have the form which provides one with a way to perform a dual transform of the superbrane models.

The worldvolume fields $Z^M(x)$ and $A(x)$ (or \mathcal{F}_{mn} and H_{mnl}) of the super-p-branes are transformed under the κ -transformations as above (see Eqs. (7)).

The κ -variation of the auxiliary fields and the Lagrange multipliers can be easily obtained from their expressions in terms of other fields whose κ -transformations are known.

To compute the κ -transformation of the actions (19) and (21) we should also know the κ -variations of the Lorentz-harmonic fields u_b^a , which are genuine worldvolume fields. However these variations are multiplied by algebraic field equations such as (16) and (20) and, therefore, they can be appropriately chosen to compensate possible terms proportional to the algebraic equations that arise from the variation of other terms. It means, in particular, that when computing $\delta_\kappa S$ we can freely use these algebraic equations and, at the same time, drop the κ -variations of these genuine worldvolume quantities if we are not interested in their specific form.

Thus by construction the actions (19) and (21) are integrals of $(p+1)$ -forms \mathcal{L}_{p+1} and one can show that their κ -variation has the following general structure [17]:

$$\delta S = \int i_\kappa d\mathcal{L}_{p+1} = \int i_\kappa \left(iE^{(-)} \gamma^{(p)} E^{(-)} - 2E^{(-)} \gamma^{(p+1)} \Delta\phi \right), \quad (22)$$

where the second term is absent from the case of the M-5-brane,

$$E^{(-)\alpha} = \frac{1}{2} E^\beta (1 - \bar{\Gamma})_{\underline{\beta}}^{\underline{\alpha}}, \quad \Delta_{\underline{\alpha}} \phi(X, \Theta) = E_{\underline{\alpha}}^M \partial_M \phi, \quad (23)$$

$\gamma_{\underline{\alpha}\underline{\beta}}^{(p)}$ and $\gamma_{\underline{\alpha}\underline{\beta}}^{(p+1)}$ are, respectively, differential p -form and $p+1$ -form constructed of worldvolume-projected target-space gamma-matrices and the tensor \mathcal{F}_{mn} (see [14, 17, 27] for details).

The fact that $i_\kappa d\mathcal{L} = 0$ and (22) is κ -invariant follows from Eqs. (7) and (10) which imply $i_\kappa E^{(-)\underline{\alpha}} = 0 = i_\kappa E^{\underline{\alpha}}$.

Note that, since the spinor parameter κ corresponds to a particular class of general variations of $\Theta(x)$, the knowledge of the κ -variation (22) of the super- p -brane action enables one to directly get equations of motion of $\Theta(x)$ as differential form equations

$$i\gamma_{\underline{\alpha}\underline{\beta}}^{(p)} E^{(-)\underline{\beta}} - \left(\frac{1}{2}(1 - \bar{\Gamma})\gamma^{(p+1)}\right)_{\underline{\alpha}}^{\underline{\beta}} \Delta_{\underline{\beta}}\phi = 0. \quad (24)$$

Let us now demonstrate how the presence of the Lorentz-harmonic fields allows one to extract in a covariant way the independent parameters of the κ -transformations (see [10, 20] for ordinary super- p -branes)¹. For this we use the $SO(1, p) \times SO(D - p - 1)$ decomposition of the spinor harmonics (18). To be concrete, consider the example of a Dirichlet 3-brane ($p=3$) in a background of type *IIB* $D = 10$ supergravity [17]. In this case the decomposition (18) of a 16×16 matrix $v_{\underline{\beta}}^{\underline{\alpha}}$ takes the form

$$v_{\underline{\beta}}^{\underline{\alpha}} = (v_{\underline{\alpha}q}^{\alpha}, \bar{v}_{\underline{\alpha}}^{\dot{\alpha}q}), \quad (25)$$

where $\alpha = 1, 2$, $\dot{\alpha} = 1, 2$ are Weyl spinor indices of $SO(1, 3)$, $q = 1, \dots, 4$ are $SO(6)$ spinor indices and bar denotes complex conjugation.

Using the exact form of the matrix $\bar{\Gamma}$ and the condition (10) one can show [17] that the following 16 complex conjugate components of the complex κ -parameter are independent:

$$\kappa_q^{\alpha}(x) = \kappa_{\underline{\beta}}^{\beta} v_{\underline{\beta}q}^{\alpha}, \quad \bar{\kappa}^{\dot{\alpha}q}(x) = \bar{\kappa}_{\underline{\beta}}^{\beta} \bar{v}_{\underline{\beta}}^{\dot{\alpha}q}. \quad (26)$$

By the use of independent parameters (such as (26)) the κ -transformations (7) can be rewritten in an irreducible form. This realization of κ -symmetry is target-space covariant since the parameters (26) are target-space scalars and carry the indices of the $SO(1, p) \times SO(D - p - 1)$ group, the first factor of which is identified with the Lorentz rotations in the tangent space of the superbrane worldvolume and the second factor corresponds to an internal local symmetry of the super- p -brane. Because of the fermionic nature of these worldvolume κ -parameters it is tempting to treat them as the parameters of n -extended local supersymmetry² in the worldvolume of the super- p -brane, and this was just a basic idea of [1], which has been fruitfully developed [2]–[11] in the framework of the doubly supersymmetric approach.

To make the local worldvolume supersymmetry manifest one should extend the worldvolume to a world supersurface parametrized by x^m and n $SO(1, p)$ -spinor variables $\eta^{\alpha q}$ all fields of the super- p -brane models becoming worldvolume superfields.

Now, the differential form structure (19) of super- p -brane actions admits of an extension to worldvolume superspace by the use of generalized action principles of the group-manifold (or rheonomic) approach [16] to supersymmetric field theories. This has been carried out for the ordinary super- p -branes [7] and the Dirichlet branes [17]. As to the M-5-brane, the presence of the term $Q_5 \wedge (da(x) - E^a v_a)$ in (21) causes problems to be solved yet to lift the M-5-brane action to worldvolume superspace, since (without some modification) this term would

¹An alternative possibility of getting irreducible covariant κ -transformations and their covariant gauge fixing has been discussed in [29].

²The number n of worldvolume supersymmetries is such that $n \times \dim Spin(1, p) = \dim Spin(1, D - 1)$.

lead to rather strong (trivializing) restrictions on worldvolume supergeometry. Thus for the time being further consideration is not applicable in full measure to the M–5–brane action (21), though final superfield equations for the superbranes which one gets as geometrical conditions of supersurface embedding are valid for the M–5–brane as well. The relation of the M–5–brane action (1) and (4) [15] and component field equations of the M–5–brane obtained from the doubly supersymmetric geometrical approach [8, 18] was established in [27].

The rheonomic approach exhibits in a vivid fashion geometrical properties of supersymmetric theories, and when the construction of conventional superfield actions for them fails the generalized action principle allows one to get the superfield description of these models. As we shall sketch below, in the case of super–p–branes the generalized action serves for getting geometrical conditions of embedding world supersurfaces into target superspaces, which completely determine the on–shell dynamics of the superbranes [6, 7, 8, 17]. However an open problem is how to extend the generalized action approach to the quantum level.

Main points of this doubly supersymmetric construction are the following.

The generalized action for superbranes has the same form as (19) but with all fields and differential forms replaced with superforms in the worldvolume superspace $\Sigma=(x^m, \eta^{\alpha q})$. The integral is taken over an arbitrary $(p+1)$ –dimensional bosonic surface $\mathcal{M}_{p+1}=(x^m, \eta^{\alpha q}(x))$ in the worldvolume superspace Σ . Thus, the surface \mathcal{M}_{p+1} itself becomes a dynamical variable, i.e. one should vary (19) also with respect to $\eta(x)$, however it turns out that this variation does not produce new equations of motion in addition to the variation with respect to other fields, the equations of motion of the latter having the same form as that obtained from the component action we started with. But now the fact that the surface \mathcal{M}_{p+1} is arbitrary and that the full set of such surfaces spans the whole worldvolume superspace makes it possible to consider these equations of motion as equations for the superforms and superfields defined in the whole worldvolume superspace Σ . The basic superfield equations thus obtained are Eqs. (16) and (24) (note that now the external differential also includes the η –derivative). Eqs. (16) and (24) tell us that induced worldvolume supervielbeins $(e^a(x, \eta), e^{\alpha q}(x, \eta))$ can be chosen as a linear combination of $E^b u_b^a \equiv E^a$ and $E^\beta v_\beta^{\alpha q} \equiv E^{\alpha q}$ [6, 8, 17]:

$$e^a = E^b (m^{-1})_b^a \quad \Rightarrow \quad E_{\alpha q}^a = 0, \quad (27)$$

$$e^{\alpha q} = E^{\alpha q} + E^a \chi_a^{\alpha q}(x, \eta), \quad (28)$$

as well as that

$$E_{\alpha q}^{(-)\alpha} \equiv \left(E_{\alpha q} \frac{1}{2} (1 - \bar{\Gamma}) \right)^\alpha = 0, \quad (29)$$

The choice of the matrix $m_b^a(x, \eta)$ is a matter of convenience and can be used to get the main spinor–spinor component of the worldvolume torsion constraints in the standard form $T_{\alpha q, \beta r}^a = i \delta_{qr} \gamma_{\alpha\beta}^a$. In this case m_b^a is constructed out of worldvolume gauge fields [8, 17, 27].

Eq. (27) together with eq. (16) implies the basic geometrodynamical condition

$$E_{\alpha q}^a = 0 \quad (30)$$

which in the doubly supersymmetric approach to super–p–branes determines the embedding of the worldvolume superspace into the target superspace. In many interesting cases such as $D = 10$ type II superstrings [4, 6] and D–branes [8, 17], and $D = 11$ branes [6, 8] the integrability conditions for (16), (27) and (28) reproduce all the equations of motion of these extended objects and also lead to torsion constraints on worldvolume supergravity [7, 17].

Note that for the D-branes and the M-5-brane the embedding conditions analogous to (16), (27), (28) and (29) were initially not derived from an action, which was not known at that time, but postulated [8] on the base of the previous knowledge of analogous conditions for ordinary super-p-branes [6, 7].

To conclude, we have demonstrated how the super-p-brane action can be reconstructed as the worldvolume integral of a differential $(p + 1)$ -form. The use of the Lorentz-harmonic variables in this formulation makes the κ -symmetry transformations to be performed with an irreducible set of fermionic parameters being worldvolume spinors. This indicates that the κ -symmetry originates from extended local supersymmetry in the worldvolume. We have shown how this worldvolume supersymmetry becomes manifest in a worldvolume superfield generalization of the super-p-brane action. The superfield equations derived from the latter are the geometrical conditions of embedding worldvolume supersurfaces swept by the superbranes in target superspaces. Thus, the approach reviewed in this article serves as a bridge between different formulations developed for describing superbrane dynamics.

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